DYNAMICAL STABILITY OF WITTEN RINGS.

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Abstract

The dynamical stability of cosmic rings, or vortons, is investigated for the particular equation of state given by the Witten bosonic model. It is found that there exists a finite range of the state parameter for which the vorton states are actually stable against dynamical perturbations. Inclusion of the electromagnetic self action into the equation of state slightly shrinks the stability region but otherwise yields no qualitative difference. If the Witten bosonic model represents a good approximation for more realistic string models, then the cosmological vorton excess problem can only be solved by assuming either that strings are formed at low energy scales or that some quantum instability may develop at a sufficient rate.

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I. INTRODUCTION

Cosmic strings [1] have been proposed as seeds for large scale structure formation [2] and as a means to reproduce temperature fluctuations in the cosmic microwave background [3] if they appear at the Grand Unified (GUT) phase transition. However, the underlying string theories in these calculations assume structureless strings, i.e. the kind for which the equation of state is that of Goto-Nambu (having their energy per unit length U equal to their tension T, both being strictly constant along the string worldsheet), so the question arises to what extent these conclusions are still valid in more complicated cases such as those proposed by Witten [4] where a charged condensate is formed in the string's core, thereby inducing a current and thus modifying the equation of state into a nondegenerate one. It has been argued [5,6] that in the latter case, equilibrium configurations (called vortons [5] or rings [6]) might exist, that would not radiate all their energy in the form of gravitational waves (which is the case for Goto-Nambu strings), and therefore contribute non negligibly to the overall matter density in the Universe. Estimates of the corresponding vorton distribution led Davis and Shellard [5] and Carter [6] to the conclusion that the stability of such states would imply a huge matter-density remnant, so that current-carrying strings appearing at the GUT phase transition would induce $\sim 10^{20}$ times the critical density, and that a premature collapse of the Universe would be avoided only if they are lightweight (being produced at energy scales at most comparable to that of the electroweak phase transition).

There are in fact two necessary ingredients for this overdensity problem. The first concerns the currents in the strings, since this is an essential requirement for the equilibrium configurations to exist: rings are centrifugally supported string loops and thus are only defined when the equation of state is nondegenerate so that there exists a prefered frame in which rotation makes sense. This issue has been addressed in a particle physics framework with the following conclusion. If the string forming GUT theory is to have as a low energy limit the standard electroweak model (a "naturalness" requirement), then the string forming fields cannot be arbitrarily decoupled from electromagnetism, and currents will appear in strings, either through quantum tunnelling [7], or by spontaneous current generation [8]. This can in fact be traced back to the nonabelian nature of the GUT model being used: since the string forming Higgs field expectation value vanishes in the string's core, some charged gauge vectors are massless there, and induce an electromagnetic instability [8] or metastability [7], depending on the coupling constant values. In both cases, currents appear in strings as a generic feature, so the problem remains. Moreover, it has also been shown [7] that even in these realistic (and therefore more complicated) models, the equation of state can be well approximated by that of the Witten bosonic model, which was recently calculated in detail [9–11]. This is the reason why this particular equation of state is used in the present work.

The second necessary requirement to have a string dominated Universe is that the equilibrium configurations be classically stable, at least dynamically. This is not an easily verified assumption, and the purpose of this work is precisely to investigate it in the framework of the Witten bosonic model, both in the neutral limit [10] case where the electromagnetic coupling is made to vanish, and in the charge coupled case, by implementing the actual value of this coupling constant on the equation of state [11], including back reaction on the constituant fields. Since this work is primarily concerned with dynamical stability, we have

not included the electromagnetic corrections either to the the equilibrium condition or to the stability constraint, these (expected stabilising) corrections being of a higher order and left for further examination [12].

This work is arranged as follow. In a first part, we recall the basic dynamical equations for a string loop, and the equilibrium condition, as well as the stability constraints [13]. This is done assuming an underlying equation of state, or, more precisely, by assuming the characteristic perturbation velocities to be given. These velocities correspond to transverse and soundlike perturbations, being expressible respectively as $c_T^2 = T/U$ and $c_L^2 = -dT/dU$. Then, a brief summary of the characteristic features of the Witten bosonic model is given and we ultimately apply the stability calculations for this model to show that a finite range of currents actually yields stable states. Since it turns out that this range is for low values of the current, or for nearly lightlike currents, we argue that most vortons are in fact stable, so that, unless quantum radiation provide some efficient way to destabilise them, they can only have been created at a low energy phase transition.

II. STABILITY OF A RING CONFIGURATION

In order to calculate the dynamical evolution of a cosmic string, we assume it to be infinitely thin and characterize the corresponding two-dimensional worldsheet by a set of two orthonormal tangent vectors u^μ and v^μ, respectively timelike and spacelike. These vectors allow an easy definition of the first and second fundamental tensors of the string's worldsheet, namely [14,15]

$$\eta^{\mu\nu} \equiv -u^\mu u^\nu + v^\mu v^\nu, \quad K_{\mu\nu}{}^\rho \equiv \eta^\sigma{}_\mu \eta^\lambda{}_\nu \nabla_\lambda \eta^\rho{}_\sigma, \tag{1}$$

the latter satisfying the Weingarten identity

$$K_{[\mu\nu]}{}^\rho = 0, \tag{2}$$

which is the integrability condition for the existence of a worldsheet containing u^μ and v^μ as tangent vectors.

Since the string is considered as infinitely thin, its stress-energy tensor $T^{\mu\nu}$ is a distribution defined on the string worldsheet only, and therefore involves in principle singular δ-functions. However, use of such distributions can be avoided [15] by working with a surface stress-energy tensor $\widetilde{T}^{\mu\nu}$, depending on the internal coordinates of the worldsheet, τ and σ say, such that the corresponding distribution valued tensor $T^{\mu\nu}$ can, if needed, be obtained from the formula

$$T^{\mu\nu}(x) = (-g)^{-1/2} \int dS_2 \widetilde{T}^{\mu\nu}(\tau, \sigma) \delta^4(x - x(\tau, \sigma)), \tag{3}$$

where $x(\tau, \sigma)$ is a generic point of the worldsheet, dS_2 the surface measure and g is the determinant of the metric $g^{\mu\nu}$ of the surrounding four dimensional space-time (which is supposed to be flat in the following analysis). Conservation of the stress-energy tensor $T^{\mu\nu}$ then implies that the surface stress tensor satisfies a corresponding worldsheet conservation law having the form [14]

$$\eta^\lambda{}_\nu \nabla_\lambda \widetilde{T}^{\mu\nu} = 0. \tag{4}$$

This is the dynamical part of the string motion. It can be further simplified by choosing the frame (u^μ, v^μ) used in Eq. (1) to be identified with the frame in which $\tilde{\mathbf{T}}$ is diagonal, with eigenvalues U and T respectively the energy per unit length and tension, in the form

$$\tilde{T}^{\mu\nu} = Uu^\mu u^\nu - Tv^\mu v^\nu. \tag{5}$$

It is convenient for computational purposes to start with u^μ and v^μ as basic independent variables whose integration determines the worldsheet as a secondary construct. In such an approch [13], the Weingarten condition (2) is not satisfied automatically but must be included as an additional dynamical equation together with Eq. (4). Subject to provision of the equation of state (see next section), Eqs. (2) and (4) provide a complete description of the string motion, expressible in terms of the transverse perturbations and the longitudinal group velocities

$$c_T = \sqrt{\frac{T}{U}} \ , \ c_L = \sqrt{-\frac{dT}{dU}}, \tag{6}$$

as the following system:

$$K_{[\mu\nu]}{}^\rho = 0 \Leftrightarrow \quad \perp_{\mu\nu} (u^\rho \nabla_\rho(v^\nu) - v^\rho \nabla_\rho(u^\nu)) = \quad 0, \tag{7}$$

$$\perp_{\mu\nu} \overline{\nabla}_\rho \tilde{T}^{\rho\nu} = \quad \perp_{\mu\nu} (u^\rho \nabla_\rho(u^\nu) - c_T^2 v^\rho \nabla_\rho(v^\nu)) = \quad 0, \tag{8}$$

$$u_\nu \overline{\nabla}_\rho \tilde{T}^{\rho\nu} = \quad Au^\rho \nabla_\rho(c_T^2) - (1 - c_T^2)u_\nu v^\rho \nabla_\rho(v^\nu) = \quad 0, \tag{9}$$

$$v_\nu \overline{\nabla}_\rho \tilde{T}^{\rho\nu} = \quad -Ac_L^2 v^\rho \nabla_\rho(c_T^2) + (1 - c_T^2)u_\nu u^\rho \nabla_\rho(v^\nu) = 0, \tag{10}$$

where $\perp_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ is the orthogonal projection operator and $A = -(c_T^2 + c_L^2)^{-1}$. The unknown quantities [16] associated with this system can be chosen as the five independent components of the tangent vectors u^μ and v^μ and c_T^2, while c_L^2 is given a priori as a function of c_T^2 by the equation of state.

Let us now more specifically turn to the case of a circular rotating string with radius R, angular speed Ω and running velocity $v = R\Omega$, for which one can express the eigenvectors in cylindrical coordinates (t, ρ, θ, z) as

$$u^\mu = \gamma(1, 0, v, 0) \quad \text{and} \quad v^\mu = \gamma(v, 0, 1, 0), \tag{11}$$

where $\gamma = 1/\sqrt{1 - v^2}$ is the Lorentz factor associated to the running velocity v. It may be shown [6,13], that the ring configuration (11) automatically satisfies the system of Eqs. (7)-(10), except for Eq. (8) which provides the simple relation (again, not taking into account the small long-range electromagnetic back reaction [17])

$$v = c_T. \tag{12}$$

Configurations for which Eq. (12) hold are the equilibrium states (vortons or rings) whose condition of stability is summarized below. This relation is in fact more general in the sense that it must hold also for any non straight equilibrium state with a static Killing vector [13].

We now consider the perturbations of the equilibrium state (12). Without loss of generality, and thanks to the symmetries of this configuration, the perturbed quantities can be taken in the form of plane waves with pulsation ω and integer angular momentum n, so that, in particular, the perturbed parameters describing the string's location read as

$$\rho = R + \delta R e^{i(\omega t - n\theta)}, \tag{13}$$

$$z = \delta z e^{i(\omega t - n\theta)}. \tag{14}$$

with δR and δz constant amplitudes small compared to the unperturbed value R. The 6 perturbed independent quantities [16] in this particular case are chosen as δu^ρ, δu^θ, δu^z, δv^ρ, δv^z and δc_T^2, out of which one can reconstruct the full perturbed ring, and in particular δR and δz, this reconstruction being consistent only in the case where the geometrical Eq. (2) is satisfied.

The equations (7) – (10) give a linear homogeneous system of six equations with six unknowns in which the azimuthal perturbations as exemplified by δu^z and δv^z decouple from the equatorial ones consisting of the other unknowns δu^ρ, δu^θ, δv^ρ and δc_T^2. The azimuthal part of this system yields the modes [16]

$$\omega \in \left\{ 0, \frac{2nv}{R(1+v^2)} \right\} \tag{15}$$

which are all stable (their imaginary part vanishing). Therefore, we shall only be concerned by the equatorial modes, given [13] as the solutions of the third degree polynomial in $\sigma \equiv \omega/\Omega$

$$v^2(1+v^2)(1-c^2v^2)\sigma^3 + 2nv^2[c^2 - v^2 - 2(1-c^2v^2)]\sigma^2 +$$
$$[4v^2(1-c^2)(n^2-1) - (1+v^2)(c^2-v^2)(n^2+1)]\sigma + 2n(c^2-v^2)(n^2-1) = 0, \tag{16}$$

with $c \equiv c_L$, and where the static mode $\sigma = 0$ has been implicitly extracted out. This equation might have complex roots, in which case the corresponding state will be unstable, with characteristic life-time $\tau^{-1} = |\mathcal{I}m\,\omega|$. A previous analysis [16], using explicit solutions of Eq. (16) given by the Cardan formulae, exhibited the stability and instability regions in the square (c_L^2, c_T^2) as well as provided analytic expressions for the imaginary part of the modes in the latter case. It was found that while approaching the corner $c_L^2 = c_T^2 = 1$ (where the Witten model is located), the zones where instabilities might develop turn into vanishingly thin surfaces so that almost any equation of state must here cross stable zones. This observation was the starting point for the following closer examination of the Witten model in this context.

III. THE WITTEN BOSONIC MODEL

The most simple model that leads to superconducting cosmic strings consists in the abelian Higgs model in which a Higgs field Φ breaks a U(1) symmetry by means of a non-vanishing vacuum expectation value $\langle |\Phi| \rangle = \eta$, thereby giving a mass to a gauge vector boson B_μ, coupled to a simplified representation of electromagnetism in which a charged scalar field Σ (representing some sort of average of all the various possible fields involved in the underlying theory) is coupled to the photon A_μ. The general Lagrangian density that describes such fields is

$$\mathcal{L} = -\frac{1}{2}|D_\mu\Phi|^2 - \frac{1}{2}|D_\mu\Sigma|^2 - \frac{1}{16\pi}H_{\mu\nu}{}^2 - \frac{1}{16\pi}F_{\mu\nu}{}^2 - \mathcal{V}(\Phi, \Sigma), \tag{17}$$

where the covariant derivatives are defined through

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$$D_\mu \Phi \equiv (\partial_\mu + iqB_\mu)\Phi, \quad D_\mu \Sigma \equiv (\partial_\mu + ieA_\mu)\Sigma, \tag{18}$$

the Maxwell tensors are

$$H_{\mu\nu} = \partial_{[\mu} B_{\nu]}, \quad F_{\mu\nu} = \partial_{[\mu} A_{\nu]}, \tag{19}$$

and the interaction potential can be taken as

$$\mathcal{V}(\Phi, \Sigma) = \frac{\lambda_\phi}{8}(|\Phi|^2 - \eta^2)^2 + f(|\Phi|^2 - \eta^2)|\Sigma|^2 + \frac{\lambda_\sigma}{4}|\Sigma|^4 + \frac{m_\sigma^2}{2}|\Sigma|^2. \tag{20}$$

This model allows vortex solutions with a Σ-condensate responsible for a change in the equation of state as we now recall.

In order to study the stability of ring configurations, it is necessary either to calculate the full field equations in a circular vortex configuration, or, assuming the thin string approximation to hold, to assume the string locally straight for the fields, calculate the various integrated (on a transverse plane) components of the stress energy tensor, and plug these back into Eq. (3). These values can afterwards be used in the previouly developed formalism, so it is this approach that we shall follow here. Thus, we shall consider a portion of straight string, which we choose to be aligned with a coordinate axis z, and, working in cylindrical coordinates, we study a Nielsen-Olesen [18] vortex solution of the form

$$\Phi = \varphi(r) \exp(iN\theta), \quad \Sigma = \sigma(r) \exp[i\psi(z, t)], \tag{21}$$

for some integer winding number N, ψ being possibly restricted to the simple form

$$\psi(z, t) = az - bt. \tag{22}$$

It can be shown [10,11] that the only arbitrary parameter for a string in this model (assuming the underlying coupling constants to be given) is the phase gradient

$$\nu = \pm\sqrt{|\partial_\mu \psi \partial^\mu \psi|}, \tag{23}$$

where the + or − sign must be chosen according to whether the conserved electromagnetic current

$$\mathcal{J}^\mu \equiv \frac{1}{e}\frac{\delta \mathcal{L}}{\delta A_\mu} = \sigma^2(\nabla^\mu \psi + eA^\mu) \tag{24}$$

is respectively spacelike or timelike. A very important point concerning this current, and the corresponding existence of centrifugally supported equilibrium states, is that it is defined even in the so-called neutral limit [10] where the electromagnetic coupling constant e is made to vanish. This model therefore accounts in particular for neutral-carrier condensates in cosmic strings, and allows an easy recognition of the fact that the long range feature of electromagnetism is essentially irrelevant in ring dynamics. On the figures explained below, curves have been plotted using both $e = 0$ and a large value for this parameter producing only minor *quantitative* corrections (exaggerated on the figures).

The other conserved quantity that is needed is the stress energy tensor

$$T^\mu_\nu = -2g^{\mu\alpha}\frac{\delta\mathcal{L}}{\delta g^{\alpha\nu}} + \delta^\mu_\nu\mathcal{L}, \tag{25}$$

which yields, in the notation of the previous section, Eq. (5)

$$U = \tilde{T}^{tt} = 2\pi\int r\,dr\,T^{tt} \tag{26}$$

and

$$T = -\tilde{T}^{zz} = -2\pi\int r\,dr\,T^{zz}. \tag{27}$$

The procedure now goes as follows: to each value of the state parameter ν corresponds a (numerically computed) field configuration which can be integrated to yield U and T, with which we calculate the velocities c_T and c_L. (More details concerning these computations can be found in particular in Refs. [10,11].) A characteristic result is shown on Fig. 1 on which are plotted these velocities as functions of the state parameter ν expressed in units of the mass of the current-carrier Σ, in the neutral limit (dashed curves) and in the case where the full electromagnetic self action has been taken into account (full curves). Again, it should be stressed that the corrections resulting from the inclusion of the electromagnetic coupling constant e affect our results only in a quantitave way, the perturbation velocity plots being almost indistinguishable in most of the parameter space.

IV. RESULTS AND CONCLUSIONS

The Witten bosonic model has several underlying parameters, all of which are supposed to be fixed by the underlying string-forming microscopic theory. For each particular string, there is only one that remains independent, namely the squared phase gradient ν^2 of the current-carrier condensate, called the state parameter, whose sign reflects the timelike or spacelike nature of the superconducting current, and whose amplitude gives, in a nontrivial way, the amplitude of the corresponding current, and the degeneracy of the stress-energy tensor. For various values of the underlying parameters, we have derived the variations of the energy per unit length U and the tension T with the state parameter, enabling us to calculate the actual values, in these models, of the perturbation velocities $c_T{}^2 = T/U$ and $c_L{}^2 = -dT/dU$, as illustrated on Fig. 1, both for vanishing electromagnetic coupling constant and with inclusion of the full self action on the string-forming fields. We have then used these values to question the stability of ringlike configurations (vortons) against azimuthal and equatorial perturbations, and we found that generic results, meaning ones roughly independently of the values of the underlying parameters (coupling constants λ_ϕ, λ_σ, q, e, masses, $\cdots$), could be drawn as exemplified in Figs. 1 and 2.

The first point, as was already emphasized in previous work (but whose implications had not yet been worked out in full detail), is that the velocity of transverse perturbation always exceeds that of longitudinal perturbations, even for very low current values, so that the first order approximation for the evolution of a superconducting string, namely that for which an effective action of the form

$$S = \text{const}\int d^2s(\nabla_\mu\psi + eA_\mu)^2, \tag{28}$$

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where the transverse degrees of freedom have been integrated out, is in fact unsatisfactory as soon as one needs to consider derivatives of the equation of state. For such purposes, the alternative action [15]

$$S = \text{const} \int d^2 s \sqrt{m^2 + (\nabla_\mu \psi + eA_\mu)^2}, \tag{29}$$

with m a constant with the dimension of mass, should preferentially be used since it reproduces most of the actual features of the Witten model, and besides, is completely integrable (in the sense that analytic solutions for the string worldsheet can be explicitly constructed [19,20]) and thus leads to much simpler equations of motion. This may be understood by stating that the crucial contribution for the derivatives involved in c_L^2 is given by the fourth order term in the phase gradient, assumed to be identically zero in the action (28), though not in Eq. (29). More precisely, it can be seen that Eq. (28) implies $c_L = 1$, whereas in fact, in the Witten model, it is not only less that the speed of light, but also less than c_T. In that sense, Eq. (29) gives the better approximation $c_L = c_T$, although stability considerations do not apply since this relation has been shown [13] to imply absolute stability against any dynamical perturbations. For currents small enough, and when the internal degrees of freedom of the strings are neglected, the approximation (28) can still be used for some purposes, but only in a more restricted range of applications than previously thought.

Now the problem is that if the longitudinal velocity had been greater than the transverse one, the ring dynamical stability issue could have been addressed far more easily since in this region of the (c_L, c_T) plane, circular configurations are always stable. However, it is less obvious what will happen in the Witten string case: the regions of instability in the (c_L, c_T) plane reduce to lines close to the $c_L = c_T = 1$ edge where the most interesting part of the Witten-model's equation of state is. Therefore, although it is not possible to show it explicitly on a plot (the number of unstable zones is in principle infinite as one approaches the $c_T = 1$ line and it was not possible to draw a clear graph), it appears that the equation of state will cross various stable and unstable regions as it goes away from the stable point $c_L = c_T = 1$ (including in particular the null current case), before ultimately reaching the unstable region. This can be shown alternatively as on Fig. 2 where the characteristic inverse life-time of the configuration, expressed in units of the angular velocity Ω of the ring, is calculated [as the larger imaginary part of the solutions of Eq. (16)] as a function of the state parameter. This figure shows explicitly the range of phase gradient where the corresponding ring states are stable. Various remarks need to be made. First, the electromagnetic correction to the equation of state yields a small correction to the actual size of the stability region, but does not modify the result qualitatively. It should be emphasized that for a realistic underlying field theory in which the mass M_ϕ of the string forming Higgs particle is expected to be much larger than m_σ, the relevant coupling constant would be so small [11,21]

$$e^2 (m_\sigma/M_\phi)^2 \sim 10^{-6} \tag{30}$$

that its effect would be imperceptible. In order to obtain an effect large enough to be visible on the figure, an artificially exaggerated value $e^2 (m_\sigma/M_\phi)^2 = 0.1$ has therefore been used. Therefore, one important conclusion that can again be drawn from these results is that electromagnetic corrections to the field equations in the string core do not significantly modify the actual dynamics of a string, so that use of the neutral limit model [10] is justified.

The last point we want to emphasize concerns the vorton excess problem. As the equilibrium condition happens to be reached primarily for quasinull currents [5], for which ν is very small, it can be argued, on the basis of our results, that most of these vortons are in fact dynamically stable since this is precisely the region where stability actually occurs. So our final conclusion is that if the Witten bosonic model is a good approximation for describing superconducting cosmic strings, then any string-forming theory will produce dynamically stable vortons which can lead to an overdensity in the Universe unless the phase transition happens at low enough energy (estimated of order 10 TeV [6]), or if quantum instabilities can develop with sufficient rate. This can also be understood as a constraint on the scheme of symmetry breaking in any string forming GUT model or on the values of its microscopic parameters.

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FIGURES

FIG. 1. Plot of the transverse and longitudinal velocities $c_T^2 = T/U$ and $c_L^2 = -dT/dU$ as functions of the phase gradient $\nu^2 = |\partial_\mu \psi \partial^\mu \psi|$ of the current carrier (in units of the inverse current-carrier mass). Dashed lines are for the neutral limit with vanishing electromagnetic coupling constant, whereas solid lines include the full electromagnetic back reaction on the underlying fields, assuming an unreasonably large value for the coupling constant e to enhance the effect.

FIG. 2. The inverse life-time $\tau^{-1} = |\mathcal{I}\mathrm{m}\,\omega|$ given by the complex roots of Eq. (16) for the Witten bosonic model, in units of the angular velocity Ω of the ring. A finite region of the state parameter space has an infinite life-time (i.e. yield stable rings), for small or quasi-lightlike currents, and this is a generic feature in this model.

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